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The Algorithm of Optimal Polynomial Extrapolation of Random Processes

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Abstract. This work deals with the modelling and prediction of the realizations of random processes in corresponding future time moments. The extrapolation algorithm of nonlinear random process for arbitrary quantity of known significances and random relations used for forecasting has been received on the basis of mathematical instrument of canonical decomposition. The received optimal solutions of the nonlinear extrapolation problem, as well as the canonical decomposition, that was use as a base for optimal solution, does not set any substantional restrictions on the class of investigated random process (liniarity, Markov processes propety, stationarity, monotonicity etc.). Theoretical results, block-diagrams for calculation procedures and the analysis of applied applications, especially for the prediction of economic indexes and parameters of technical devices, are under discussions.

Keywords: random process, canonical decomposition, extrapolation algorithm.

1 Introduction

A solution of problems concerning modelling and prediction of realizations of random processes in corresponding future time moments is an actual direction of modern scientific researches, as most of the physical, technical, economic and other real processes have a stochastic character. There are a large number of different methods of extrapolation of random processes taking into account different real assumptions. Presently the forecast theory, taking into account an exceptional meaningfulness of

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problem, is constantly complemented by new algorithms that extend the class of investigated random processes and conditions for problem solutions.

2 Problem Statement

Let a random process $X(t)$ in the fixed set of points $t_i, i=\overline{1,I}$ be fully defined by means of the digitized moment functions:

$$M\left[X^{\nu}(i)\right], M\left[X^{\nu}(i)X^{\mu}(j)\right],\ \ t_i, t_j=\overline{1,I};\ \ \nu,\mu=\overline{1,N}.$$

For the known values $x^{\mu}(j),\ t_j=\overline{1,k},\ \mu=\overline{1,N}$ of the investigated realization $x(t)$ of the random process $X(t)$ it is necessary to forecast the values of this realization in future moments of time $t_i,\ i=\overline{k+1,I}$.

In [1] a universal solution of the problem of extrapolation of a realization of the random process has been received in the following recurrent form

$$m_x^{(\mu)}(i)=\begin{cases} M\left[X(i)\right],\ \mu=0,\ \mathrm{i}=\overline{1,\mathrm{I}} \\ m_x^{(\mu-1)}(i)+\left[x(\mu)-m_x^{(\mu-1)}(\mu)\right]\varphi_{\mu}(i),\ \mu=\overline{1,\mathrm{k}}\ ,\ \mathrm{i}=\overline{\mu+1,\mathrm{I}} \end{cases} \tag{1}$$

or in a vivid form

$$m_x^{(k)}(i)=M\left[X(i)\right]+\sum_{j=1}^{k}\left(x(\mu)-M\left[x(\mu)\right]\right)f_{\mu}^{(k)}(i),\ i=\overline{k+1,I}; \tag{2}$$

$$f_{\mu}^{(k)}(i)=\begin{cases} f_{\mu}^{(k-1)}(i)-f_{\mu}^{(k-1)}(k)\varphi_k(i),\ \ \mu\leq k-1 \\ \varphi_k(i),\ \mu=k \end{cases}. \tag{3}$$

where $\varphi_{\mu}(i),\ \ \mu=\overline{1,k}$ - are coordinate functions of a canonical expansion [1,2] of the random process $X(t)$, based on the points $t_i,\ i=\overline{1,I}$:

$$X(i)=\sum_{\nu=1}^{i}V_{\nu}\varphi_{\nu}(i),\ i=\overline{1,I}. \tag{4}$$

The parameters of a canonical expansion (4) are defined by the following recurrent relations:

$$V_i=X(i)-\sum_{\nu=1}^{i-1}V_{\nu}\varphi_{\nu}(i),\ i=\overline{1,I}\ ; \tag{5}$$

$$D_V(i) = M\left[V_v^2\right] = M\left[X^2(i)\right] - M^2\left[X(i)\right] - \sum_{\nu=1}^{i-1} D_V(\nu)\varphi_\nu^2(i)\,,\ \ i=\overline{1,I}\,; \tag{6}$$

$$\varphi_\nu(i) = \frac{1}{D_V(\nu)}\left[M\left[X(\nu)X(i)\right] - M\left[X(\nu)\right]M\left[X(i)\right] - \sum_{j=1}^{\nu-1} D_V(j)\varphi_j(\nu)\varphi_j(i)\right], \tag{7}$$
$$\nu=\overline{1,I}\,,\ i=\overline{\nu,I}.$$

Expressions (1), (2), within the framework of the linear approximation, determine a posterior mathematical expected value of the random process $X(t)$ under the condition that $X(\mu)=x(\mu),\ \mu=\overline{1,k},$ which in other words give the undisplaced estimation $m_x^{(k)}(i),\ i=\overline{k+1,I}$ of future values $x(i),\ i=\overline{k+1,I}$ of the extrapolated realization, and provide a minimum of the mean-square error of the extrapolation $E_x^{(k)}(i)$, which is equal to the dispersion $D_x^{(k)}(i)$ of a posteriori random process $X^{(k)}(i)$, where in particular:

$$E_x^{(k)}(i) = M\left[\left|m_x^{(k)}(i) - X(i)\right|^2\right],\ \ i=\overline{k+1,I}; \tag{8}$$

$$E_x^{(k)}(i) = D_x^{(k)}(i) = \sum_{\nu=k+1}^{i} D_V(\nu)\varphi_\nu^2(i),\ \ i=\overline{k+1,I}; \tag{9}$$

$$X^{(k)}(i) = X(i\,/\,x(j),\ j=\overline{1,k}) = m_x^{(k)}(i) + \sum_{\nu=k+1}^{i} V_\nu\varphi_\nu(i),\ \ i=\overline{1,I}. \tag{10}$$

Thus in (1) and (2) probabilistic connections of higher-order $M[X^\nu(i)X^\mu(j)],\ \nu+\mu\geq 3$ of the random process $X(t)$ are not used and as result this limits exactness of the extrapolation. The removal of the indicated defect is possible using the forecast algorithm on the base of the canonical expansion [5,6] in which the information about the investigated process is fully considered in a discrete set of points $t_i,\ i=\overline{1,I}$:

$$X(i) = M\left[X(i)\right] + \sum_{\nu=1}^{i}\sum_{\lambda=1}^{N} W_\nu^{(\lambda)}\beta_{1\nu}^{(\lambda)}(i),\ i=\overline{1,I}. \tag{11}$$

The elements of the canonical expansion (11) are defined by the following recurrent relations:

$$W_\nu^{(\lambda)} = X^\lambda(\nu) - M\left[X^\lambda(\nu)\right] - \sum_{\mu=1}^{\nu-1}\sum_{j=1}^{N} W_\mu^{(j)}\beta_{\lambda\mu}^{(j)}(\nu) - \sum_{j=1}^{\lambda-1} W_\nu^{(j)}\beta_{\lambda\nu}^{(j)}(\nu),\ \nu=\overline{1,I}; \tag{12}$$

$$D_{\lambda}(\nu)=M\left[\left\{W_{\nu}^{(\lambda)}\right\}^{2}\right]=M\left[X^{2\lambda}(\nu)\right]-M^{2}\left[X^{\lambda}(\nu)\right]-$$
$$-\sum_{\mu=1}^{\nu-1}\sum_{j=1}^{N}D_{j}(\mu)\left\{\beta_{\lambda\mu}^{(j)}(\nu)\right\}^{2}-\sum_{j=1}^{\lambda-1}D_{j}(\nu)\left\{\beta_{\lambda\nu}^{(j)}(\nu)\right\}^{2},\ \nu=\overline{1,I}; \tag{13}$$

$$\beta_{h\nu}^{(\lambda)}(i)=\frac{M\left[W_{\nu}^{(\lambda)}\left(X^{h}(i)-M\left[X^{h}(i)\right]\right)\right]}{M\left[\left\{W_{\nu}^{(\lambda)}\right\}^{2}\right]}=\frac{1}{D_{\lambda}(\nu)}(M\left[X^{\lambda}(\nu)X^{h}(i)\right]-$$
$$-M\left[X^{\lambda}(\nu)\right]M\left[X^{h}(i)\right]-\sum_{\mu=1}^{\nu-1}\sum_{j=1}^{N}D_{j}(\mu)\beta_{\lambda\mu}^{(j)}(\nu)\beta_{h\mu}^{(j)}(i)- \tag{14}$$
$$-\sum_{j=1}^{\lambda-1}D_{j}(\nu)\beta_{\lambda\nu}^{(j)}(\nu)\beta_{h\nu}^{(j)}(i),\ \lambda=\overline{1,h},\ \nu=\overline{1,i},\ h=\overline{1,N}.$$

In the canonical expansion (11) the random process $X(t)$ in the investigated row of points is presented by means of N massives $\left\{W^{(\lambda)}\right\}$, $\lambda=\overline{1,N}$ of the uncorrelated centered random coefficients $W_{i}^{(\lambda)}$, $i=\overline{1,I}$. These coefficients $W_{i}^{(\lambda)}$ contain information about the values $X^{\lambda}(i)$, $\lambda=\overline{1,N}$, $i=\overline{1,I}$, and coordinate functions $\beta_{h\nu}^{(\lambda)}(i)$, $\lambda,h=\overline{1,N}$; $\nu,i=\overline{1,I}$ describe probabilistic connections of the order $\lambda+h$ between the sections of the random process in discrete moments of time t_{ν} and t_{i}, $\nu,i=\overline{1,I}$.

Block-diagram of the procedure for calculating the parameters of the canonical decomposition is shown in Fig. 1.

We suppose that the value $x(1)$ of the process $X(t)$ at the point t_1 is known, as a result of measuring. Consequently, the values are known:

$$w_{1}^{(\lambda)}=x^{\lambda}(1)-M\left[X^{\lambda}(1)\right]-\sum_{j=1}^{\lambda-1}w_{1}^{(j)}\beta_{1\nu}^{(j)}(1),\ \nu=\overline{1,I} \tag{15}$$

for the set of coefficients $W_{1}^{(\lambda)}$, $\lambda=\overline{1,N}$.

The substitution of the value $w_{1}^{(1)}$ in the equation (11) allows us to get a polynominal canonical expansion of a posteriori random process $X^{(1,1)}(i)=X(i/x_{1}(1))$:

$$X^{(1,1)}(i)=X(i/x(1))=M\left[X(i)\right]+\left(x(1)-M\left[X(1)\right]\right)\beta_{11}^{(1)}(i)+$$
$$+\sum_{\lambda=2}^{N}W_{1}^{(\lambda)}\beta_{11}^{(\lambda)}(i)+\sum_{\nu=2}^{i}\sum_{\lambda=1}^{N}W_{\nu}^{(\lambda)}\beta_{1\nu}^{(\lambda)}(i),\ i=\overline{1,I}. \tag{16}$$

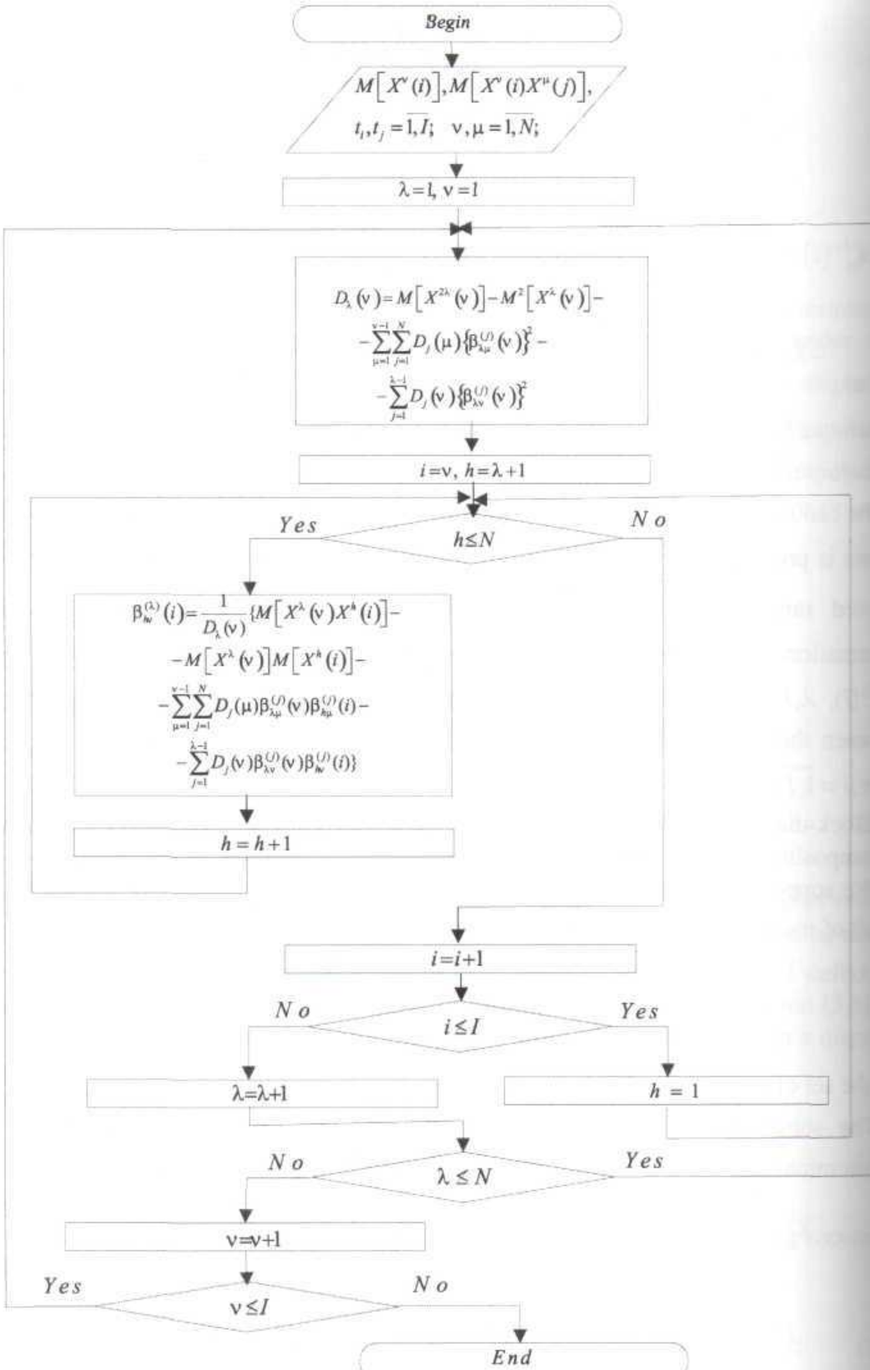

Fig. 1. Block-diagram of the procedure for calculating the parameters of the canonical decomposition

Applying expected value operation to (16) gives the optimal (by the minimum mean-square error of extrapolation criterion) estimation of future values of the random process $X(t)$ under the condition that for the determination of this estimation only value $x(1)$ is used:

$$m_x^{(1,1)}(1,i) = M\left[X(i/x(1))\right] = M\left[X(i)\right] + \left(x(1) - M\left[X(1)\right]\right)\beta_{11}^{(1)}(i),\ i=\overline{1,I}. \tag{17}$$

Considering that coordinate functions $\beta_{h\nu}^{(\lambda)}(i),\ \lambda,h=\overline{1,N};\ \nu,i=\overline{1,I}$ are determined by the minimum mean-square error of the approximation in the intervals between arbitrary values $X^{\lambda}(\nu)$ and $X^{h}(i)$, expression (17) can be generalized in case of the prediction of higher order parameters $x^{h}(i),\ h=\overline{1,N},\ i=\overline{2,I}$:

$$m_x^{(1,1)}(h,i) = M\left[X^{h}(i/x(1))\right] = M\left[X^{h}(i)\right] + \left(x(1) - M\left[X(1)\right]\right)\beta_{h1}^{(1)}(i),\ i=\overline{1,I}. \tag{18}$$

The usage of $w_1^{(2)}$ in (16) gives a canonical expansion of a posteriori process $\{X^{(1,2)}\} = X(i/x(1),x(1)^2)$:

$$X^{(1,2)}(i) = X(i/x(1),x(1)^2) = M\left[X(i)\right] + \left(x(1) - M\left[X(1)\right]\right)\beta_{11}^{(1)}(i) + \tag{19}$$

$$+\left(x^2(1) - \left(x(1) - M\left[X(1)\right]\right)\beta_{21}^{(1)}(1)\right)\beta_{11}^{(2)}(1) + \sum_{\lambda=3}^{N} W_1^{(\lambda)}\beta_{11}^{(\lambda)}(i) + \sum_{\nu=2}^{i}\sum_{\lambda=1}^{N} W_\nu^{(\lambda)}\beta_{1\nu}^{(\lambda)}(i),\ i=\overline{1,I}.$$

Applying the operation of evaluation of the mathematical expection, that uses expression (18), to the equation (19), we receive a model of extrapolation of the investigated realization of the random process on two values $x_1(1), x_1(1)^2$:

$$m_x^{(1,2)}(h,i) = M\left[X^{h}(i/x(1),x(1)^2)\right] = m_x^{(1,1)}(h,i) + \left[x^2(1) - m_x^{(1,1)}(2,i)\right]\beta_{11}^{(2)}(1),\ i=\overline{1,I}. \tag{20}$$

This generalization of the approach allows to pattern the algorithm of prediction for the arbitrary number of the known values $x^{\mu}(j),\ t_j=\overline{1,k},\ \mu=\overline{1,N}$:

$$m_x^{(\mu,l)}(h,i) = \begin{cases} M\left[X^{h}(i)\right], \mu=0 \\ m_x^{(\mu,l-1)}(h,i) + \left(x^{l}(\mu) - m_x^{(\mu,l-1)}(l,\mu)\right)\varphi_{h\mu}^{(l)}(i),\ l\neq 1 \\ m_x^{(\mu-1,N)}(h,i) + \left(x^{l}(\mu) - m_x^{(\mu-1,N)}(l,\mu)\right)\varphi_{h\mu}^{(l)}(i),\ l=1 \end{cases}. \tag{21}$$

The parameter $m_x^{(\mu,l)}(h,i) = M\left[X^{h}(i)/x^{\nu}(j),\ j=\overline{1,\mu-1};\ \nu=\overline{1,N};\ x^{\nu}(\mu), \nu=\overline{1,l}\right]$ for $h=1,\ l=N,\ \mu=k$, calculated using (21), is the undisplaced optimal estimation $m_x^{(k,N)}(1,i)$ of the future value $x(i), i=\overline{k+1,I}$, where, to determine this estimation, values $x^{\nu}(j)$, $\nu=\overline{1,N}$, $j=\overline{1,k}$ are used, i.e. the results of measuring of the random process $X(t)$ in points t_j, $j=\overline{1,k}$ are known.

Block-diagram of the calculation procedure of the future values of the random process by the algorithm (21) is shown in Fig. 2.

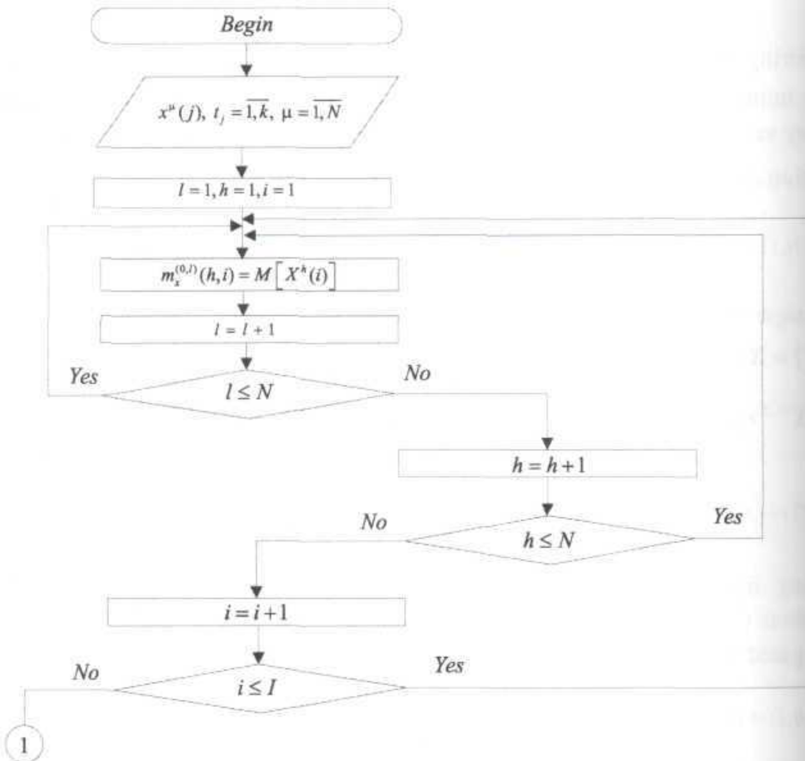

Fig. 2. Block-diagram of the calculation procedure of the future values of the random process by algorithm (21)

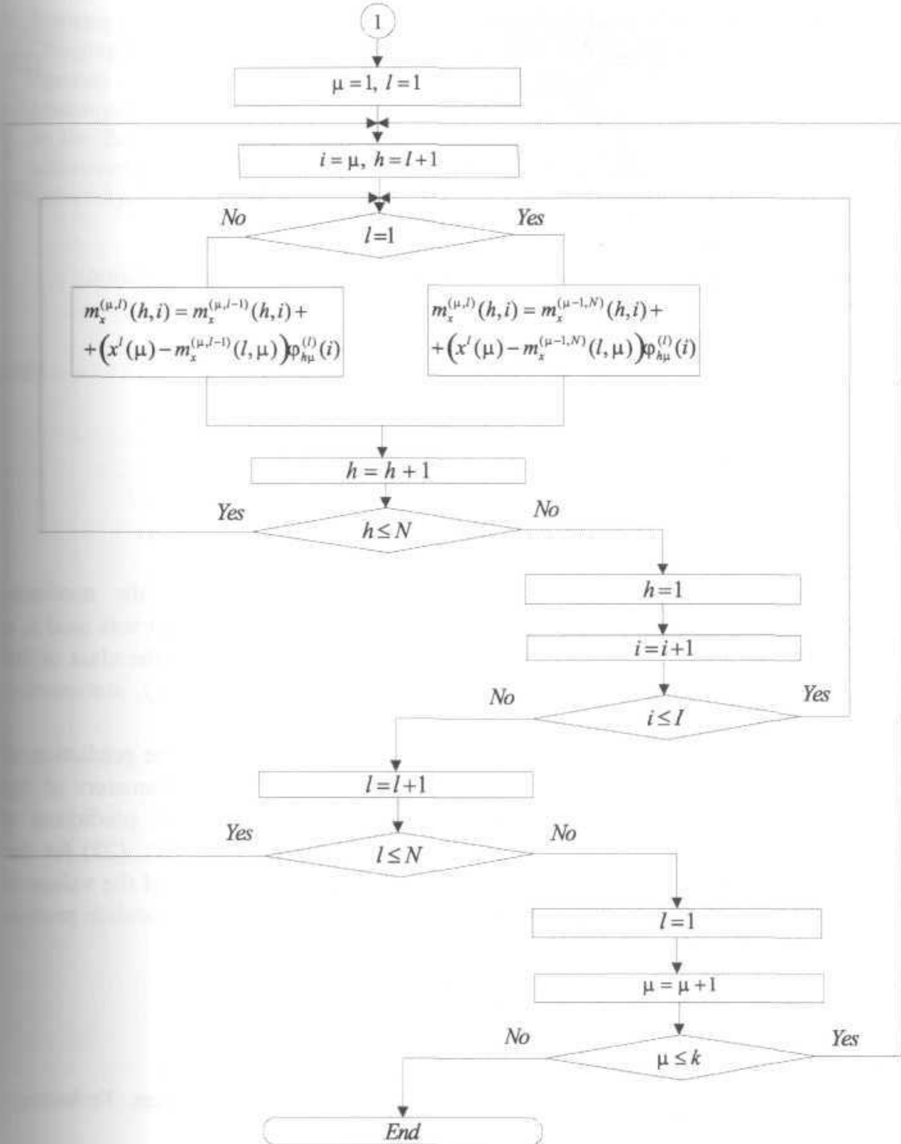

Fig. 2. *(continued)*

Expression (21) for the estimation of $m_x^{(k,N)}(1,i)$ can be transformed to the next simple form:

$$m_x^{(k,N)}(1,i) = M\left[X(i)\right] + \sum_{j=1}^{k}\sum_{\nu=1}^{N}\left(x^{\nu}(j) - M\left[x^{\nu}(j)\right]\right) S_{((j-1)N+\nu)}^{(kN)}((i-1)N+1), \tag{22}$$

$$\text{where } S_{\lambda}^{(\alpha)}(\xi)=\begin{cases} S_{\lambda}^{(\alpha-1)}(\xi)-S_{\lambda}^{(\alpha-1)}(\alpha)\gamma_{k}(i), & \lambda\leq\alpha\text{-}1 \\ \gamma_{\alpha}(\xi), & \lambda=\alpha \end{cases}; \qquad (23)$$

$$\gamma_{\alpha}(\xi)=\begin{cases} \beta_{1,[\alpha/N]+1}^{(\mathrm{mod}_{N-1}(\alpha))}\left(\left[\alpha/N\right]+1\right), & for\ \xi\leq kN \\ \beta_{1,[\alpha/N]+1}^{(\mathrm{mod}_{N-1}(\alpha))}(i), & if\ \xi=(i\text{-}1)N+1 \end{cases}. \qquad (24)$$

Thus, the mean –square error of extrapolation is determined by the expression:

$$M\left[\left\{X(i/x^{\nu}(j),\, \nu=\overline{1,N},\, j=\overline{1,k})-m_{x}^{(k,N)}(1,i)\right\}^{2}\right]=M\left[X^{2}(i)\right]-M^{2}\left[X(i)\right]- \\ -\sum_{j=1}^{k}\sum_{\nu=1}^{N}M\left[(W_{j}^{(\nu)})^{2}\right]\left\{\beta_{j}^{(\nu)}(i)\right\}^{2},\ i=\overline{k+1,I}. \qquad (25)$$

3 Conclusions

It should be noted that received optimal solution (21), (22) of the nonlinear extrapolation problem, as well as the canonical decomposition (11), that was used as a base for optimal solution, does not set any substantial restrictions on the class of the investigated random processes (linearity, Markov processes property, stationarity, monotonicity etc.).

Especially relevant is the application of algorithm (21), (22) for the prediction of the economic indexes (planning of income, gross revenue) and parameters of the technical devices (estimation of the state at future moments of time, prediction of probability of no-failure operation). The usage of the algorithm (21), (22) for the above-mentioned forecast problems is based on the fact that changes of the values of the predicted parameters are realizations of non-stationary, nonlinear random process with a considerable aftereffect.

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