



RESEARCH

An improved theory of constraints

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Abstract

Purpose – The primary aim in this paper is to develop and demonstrate a theory of constraints (TOC) model in which constraint resource prevents the throughput of the organization.

Design/methodology/approach – In this paper, the authors propose an integrated model by combining Laplace criterion and TOC into a single evaluation model in a multiproduct constraint resource environment. A case study is illustrated to demonstrate the effectiveness of this model. The outsourcing decision model compares three alternatives: standard cost accounting, standard theory-of-constraints, and our own solution.

Findings – The numerical results show that the new approach is superior to Standard cost accounting and Theory of Constraints and presents a more realistic state of optimum allocation of resources and measures the performance of the model.

Research limitations/implications – This research is limited to the production processes that do not have multiple constraints.

Originality/value – This is the first time that the integrated model comprising of Laplace-TOC model has been used to maximize the product throughput. Instead of calculating \$return per constraint minute, this method decides the priority of product that maximizes the product throughput in the constraint resource environment. It makes a significant contribution to the manufacturing Organization where one can compare the financial performance of the Organization by selecting the right decision model.

Keywords Standard costs, Outsourcing

Paper type Case study

Introduction

The constraint resources are characterized by market demand in excess of the organization's production capacity. The decision to outsource is a major strategy base for most companies, since it involves cost savings against the consequences of loss in control over the product or service. This study investigates the case where market demand exceeds the company's capacity to manufacture.

Since different models provide radically different answers to the outsourcing problem we compare three alternatives: standard cost accounting, standard theory-of-constraints (TOC; Goldratt, 1988, Fox, 1988, Ronen and Starr, 1990) and our own model.

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This paper proposes a model for evaluating the appropriateness of adopting new approach. The proposed model is based on Laplace criterion which contributes to the evaluation of the proposed model before its implementation. In recent years, many authors have written about the shortcomings of Standard accounting procedure (Johnson, 1991, Mehra *et al.*, 2005). Many authors stressed on throughput accounting (Goldratt, 1988, Dugdale and Jones, 1996, Srikant and Robertson, 1995, Srikant and Umble, 1997). Kee (1995) and Baxendale and Gupta (1998) worked on TOC with traditional cost accounting and activity based cost management. Spencer and Cox (1995) focused on software optimized production technology (OPT). Of the new management philosophies established in recent decades, the Theory of Constraints (TOC), which was developed by Goldratt at the beginning of 1980 s, plays a vital role. Its main purpose is to identify, analyze and eliminate those constraints that restrict a firm's value adding process (Goldratt and Fox, 1986). As a tool for product mix decisions, the TOC (Lockmy and Cox, 1994) based approach is often used alternatively (or parallel) to optimization tools, such as the contribution margin per constraint unit method or linear programming (LP) approaches. It is the aim of this paper to compare a new approach with the other existing model and provide more insight into the constraint resources.

There are several works by researcher on outsourcing problems. But no researchers studied the performance aspect of Laplace criterion in analyzing constraint resources. While analyzing constraint resources many researchers considered three decision models: Standard cost accounting, standard TOC and LP as base model. Ray *et al.* (2007) compares these three models with the following models: LP enhancement of goal programming technique and AHP-TOC-LP model and identify the optimum model for outsourcing. Soren *et al.* (2005) assessed quality of the TOC based approach which generate good or even optimal solutions with different results particularly when compared with other product mix decision tools. Low (1992), Luebbe and Finch (1992), Patterson (1992), Boyd and Cox (2002), and Mabin and Davies (2003) show by means of numerical results of situations with one constraint that the TOC-based approach leads to an optimal solution. Lee and Plenert (1993) analyze a slightly modified form of product mix decision, in which the launch of product under one binding constraint is analyzed. With regard to Lee and Plenert (1993), and Posnack (1994) claims that their TOC based approach was incorrect. He concludes that TOC based approach is preferable to the integer LP approach. Luebbe and Finch (1992) come to the conclusion that TOC based approach is superior to LP approach. Contrarily, Balkrishnan and Cheng (2000) explain with a small modification to the example data of Luebbe and Finch (1992) that the LP approach is superior to TOC based approach when dealing with several binding constraints. Umble *et al.* (2006) criticized the traditional cost accounting and stressed the throughput accounting. Coman and Ronen (2000), Chakrabarty *et al.* (2006), Campbell (1995), Salafatinos (1995), Gupta *et al.* (1997), and Gupta (2001) studied the performance measurement aspect of TOC with standard cost accounting and activity based cost management. Plenert (1992) found that when multiple constrained resources exist, LP is an optimum-planning tool than TOC.

The present study outlines Laplace criterion (Taha, 2006) representing a compromise between the optimistic and the pessimistic approach to decision making under uncertainty and rank the order of the product for manufacturing cost analysis.

Computation of the degree of relative importance for technical requirements is made through Laplace criterion.

This study analyzes the case where demand exceeds company's capacity to manufacture and application of Laplace criterion in the constraint resources. This evaluation technique requires raw material cost, hourly rate, selling price, demand, workflow, working time, etc.

The following section presents, a brief review of Laplace criterion, the proposed methodology and the case study using the proposed model. The last section compares three alternatives: standard cost accounting, standard TOC (Goldratt, 1988, Fox, 1988), and our model and scope of further work.

Notation

Let us introduce the following items:

- i – product index;
- j – resource index;
- P_i – market price;
- R_i – raw material cost;
- D_i – demand;
- DM – decision matrix;
- C_i – contractor's price;
- OE – operating expenses;
- X_i – units of product i produced;
- P_j – maximum available time; and
- CM_i – contribution margin of product i .

Laplace criterion

Decision making under uncertainty, as under risk, involves alternative decisions whose payoffs depends on the states of nature. The Laplace criterion is based on the principle of insufficient reason (Taha, 2006). The alternative decision are evaluated using the optimistic assumption that all states are equally likely to occur, that is, $P\{s_1\} = P\{s_2\} = \dots = P\{s_n\} = 1/n$. The payoff of a decision problem with m alternative actions and n states of nature can be represented as follows:

The elements a_k represents action k , and the elements s_l represents state of nature l . The payoff or outcome associated with action a_k and state s_l is $\nu(a_k, s_l)$.

	S_1	S_2	...	S_n
a_1	$\nu(a_1, s_1)$	$\nu(a_1, s_2)$...	$\nu(a_1, s_n)$
a_2	$\nu(a_2, s_1)$	$\nu(a_2, s_2)$...	$\nu(a_2, s_n)$
\vdots	\vdots	\vdots	...	\vdots
a_n	$\nu(a_n, s_1)$	$\nu(a_n, s_2)$...	$\nu(a_n, s_n)$

The best alternative is the one that yields:

$$\max_{a_k} \left\{ \frac{1}{n} \sum_{l=1}^n \nu(a_k, s_l) \right\}$$

Proposed methodology

The following methodology has been developed:

- (1) The in house contribution margin (CM) of each product is calculated by subtracting raw material cost from selling price. Contribution margin, $CM_i = P_i - R_i, i = 1..n$.
- (2) The contractor's contribution margin of each product i is calculated by subtracting supplier's price from selling price of the product, i.e. $CM_i = P_i - C_i$.
- (3) Develop the decision matrix as follows:

$$DM = \begin{pmatrix} a_{11} & a_{12} \\ \vdots & \vdots \\ a_{i1} & a_{i2} \\ \vdots & \vdots \\ a_{na} & a_{n2} \end{pmatrix}$$

where a_{i1} and a_{i2} are the in house throughput and contractor's throughput.

- (4) Calculate the priorities for each of the products by taking average of each row (Laplace criterion).
- (5) Identify the constraint resource.
- (6) Rank the products using the normalized weights of each product.
- (7) Calculate the time left on the bottleneck as follows:

$$\text{time left} = \text{available time} - \text{time used}$$
- (8) Whether dominant bottleneck's capacity is exhausted or there is insufficient capacity remaining to produce another unit of product.
- (9) If answer to the step 8 is 'yes' then determinations of profit and comparisons of said profit to that with other model.

A case study

A south-east Asia based manufacturing firm produces three products P, Q, R using four resources R_1, R_2, R_3 and R_4 . Owing to its limited resource company wants to outsource some product to subcontractor. The facilities are available five days a week for one shift consisting of eight hours per day. So total working minute available $5 \times 8 \times 60 = 2,400$ min. The flow layout (Figure 1) of the product processing describe each of the three product incorporates two of four raw material 1, 2, 3 and 4. The cost of each raw material is \$20. The market price of the products P, Q and R are \$120, \$160, and \$200, respectively. The customer demand is 100 units of each one of the three products. Contractors selling price to the company for P, Q, R are \$100, \$120, and \$190, respectively. These prices include the cost of raw materials (Table I).

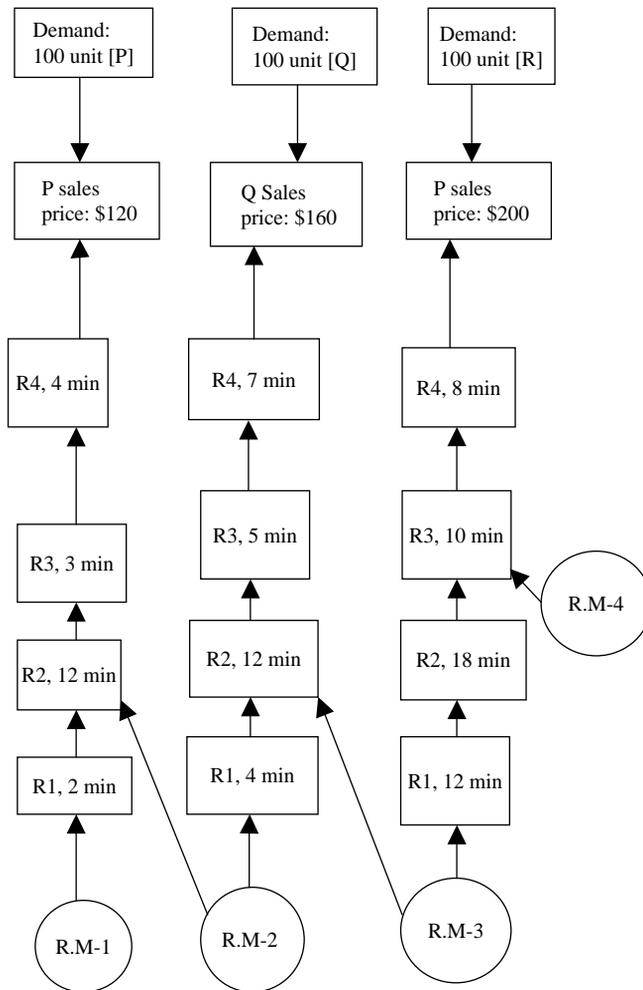


Figure 1.
Resource time per product in minutes

Resource	P	Q	R	Minutes/week	Utilization ratio, demand/2,400 (percent)
R_1	2	4	12	1,800	75
R_2	12	12	18	4,200	175
R_3	3	5	10	1,800	75
R_4	4	7	8	1,900	79.16
	21	28	48		

Table I.
Resource per product in minute

Standard accounting

Standard accounting procedure decides those products that are more profitable per production time unit. To calculate the cost of each product the company's operating expenses \$10,000 are divided by the number of resources and the maximum time limit

per week. Thus, the cost of every minute worked at any resources equal $\$10000/4/2400 = \1.041

The cost of each product is the sum of its raw material cost and total working minutes at all three resources multiplied by the company's minute rate (\$1.041). The contribution margin per product is the difference between the selling price and the cost. Product profit per minute is determine by dividing the product profit by total work minute as follows: \$2.768 for P, \$2.173 for Q and \$1.250 for R. P is the most preferred product to manufacture since its contribution per work minute is highest, followed by Q, making R the least preferred to manufacture. Hence, the constraint resource R_2 will manufacture 100 units of product P, 100 units of product Q and but due to unavailability of time no units of product R. The throughput of each manufactured product is the difference between product selling price and raw material cost. The calculated throughput is multiplied by the numbers of unit manufactured. The throughput of each contracted unit is calculated as its selling price less the price paid to the contractor. Finally, total product throughput (Table II) is the summation of manufactured throughput and contracted throughput.

Standard theory-of-constraints

TOC decides product throughput per unit of working time at the bottleneck. We divide the throughput by the bottleneck resource R_2 's time. Product Q has the highest throughput per constraint minutes, i.e. \$10 per constraint minute and is therefore the first preference to manufacture. Hence, we manufacture 100 units. Product R with a throughput per constraint minute ratio of \$8.88 per constraint minute is next (Table III). Due to R_2 's constraint capacity we only manufacture 66 units and outsource 34 units. Product P's throughput per constraint minute ratio is the lowest, i.e. \$6.66 per constraint minute and since no more manufacturing capacity is available we outsource the full 100 units.

Proposed approach

- (1) The in house contribution margin is calculated as shown in the Table IV.
- (2) The contractor's contribution margin is calculated as shown in Table V.

Product	P	Q	R
Total work min./product	21	28	48
Product cost: time + raw material cost	\$61.86	\$69.14	\$89.96
Product selling price	\$120	\$130	\$200
Product profit	\$58.14	\$60.86	\$60.04
Product profit/min	\$2.768	\$2.173	\$1.250
Demand	100	100	100
Unit to manufacture	100	100	0
Unit to contractor	0	0	100
Throughput per manufactured unit	\$80	\$120	\$160
Throughput per contracted unit	\$20	\$40	\$10
Product throughput from manufactured quantity	\$8,000	\$12,000	0
Product throughput from contracted quantity	0	0	\$1,000
Total facility throughput		\$21,000	
Operating expenses		\$10,000	
Net profit		\$11,000	

Table II.
Standard accounting
analysis

Component	P	Q	R
Selling price per unit	\$120	\$160	\$200
Raw material cost per unit	\$40	\$40	\$40
Product throughput	\$80	\$120	\$160
Constraint resource "R ₂ " min	12	12	18
Throughput/constraint R ₂ min	\$80/12 min = \$6.66/min	\$120/12 min = \$10/min	\$160/18min = \$8.88/min
Demand in units	100	100	100
Unit to manufacture	0	100	66
Unit to contractor	100	0	34
Throughput per manufactured unit	\$80	\$120	\$160
Throughput per contracted unit	\$20	\$40	\$10
Product throughput for manufactured unit		\$12,000	\$10,560
Product throughput for contracted unit	\$2,000	0	\$340
Total facility throughput		\$24,900	
Operating expenses		\$10,000	
Net profit		\$14,900	

Table III.
Standard theory-of-constraints

Product	Demand (unit)	Selling price (\$)	Raw material cost (\$)	CM (\$)
P	100	120	40	80
Q	100	160	40	120
R	100	200	40	160

Table IV.
In house contribution margin

Product	Demand (unit)	Selling price (\$)	Contractor's price (\$)	CM (\$)
P	100	120	100	20
Q	100	160	120	40
R	100	200	190	10

Table V.
Contractor's contribution margin

- (3) The decision matrix of the problem is as shown in Table VI.
- (4) Table VII shows the calculated value of row minimum and row maximum.
- (5) Resource R₂ is the bottleneck because it is most overloaded.
- (6) Table VIII shows the priorities of each product. The production priority should be developed in view of each bottleneck. As shown in table resource R₂ is the bottleneck. Decision maker assigns score 0.395 to the product R. In other words, product R has the highest priority and so on.
- (7) The constraint resource at resource centre R₂ makes us to produce 100 units of R, 50 units of Q and none or 0 of P as shown in the following table. There are

Product	Contribution margin (in house)	Contribution margin (contracted)
P	80	20
Q	120	40
R	160	10

Table VI.
Decision matrix-1

2,400 min available for resource 2. Producing 100 Q will leave $2,400 - 18 \times 100 = 600$ min. Producing 50 units will leave $600 - 50 \times 12 = 0$ min (Table IX).

- (8) Now no time is available for producing another unit of product.
- (9) The net profit is obtained by subtracting the plant's operating expenses from the total throughput. In house throughput for product R = $100 \times \$160 = \$16,000$. The throughput for product Q = $50 \times \$120 = \$6,000$ and P = 0, respectively. Contracted throughput for product Q = $50 \times \$40 = \$2,000$, R = 0 and P = $100 \times \$20 = \$2,000$. The plant's operating expenses = \$10,000. Therefore, the total net profit = \$16,000.

Conclusion and discussion

In recent years different approaches have been proposed to deal with outsourcing problem. An efficient assessment system is essential for appropriate model selection. In this paper Laplace criterion is used to assess the performance of each product. The result was compared (Table X) with standard accounting, TOC model. Out of these three models the standard accounting solution is inferior to standard TOC solution due

Table VII.
Decision matrix-2

Product	Contribution margin (in house)	Contribution margin (contracted)	Average
P	80	20	50
Q	120	40	80
R	160	10	85

Table VIII.
Product priorities

Product	Contribution margin (in house)	Contribution margin (contracted)	Average	Product priorities
P	80	20	50	0.232
Q	120	40	80	0.372
R	160	10	85	0.395

Table IX.
Resource load analysis

Product	Market demand	Product quantity	Weight age	Time (minutes)	Used (minutes)	Left (minutes)
R	100	100	0.395	18	1,800	600
Q	100	50	0.372	12	600	0
P	100	0	0.232	12	0	0

Table X.
Summery of three methodologies

Components	Standard accounting	TOC	Revised TOC
P-manufactured	100	0	0
Q-manufactured	100	100	50
R-manufactured	0	66	100
P-Outsourced Quantity	0	100	100
Q-outsourced quantity	0	0	50
R-outsourced quantity	100	34	0
Net profit	\$11,000	\$14,900	\$16,000

to its treatment of equal weight age to all resources. Such an assumption is not realistic and only applies when all resources have identical utilization ratio. The standard TOC solution is also inferior to improved theory of constraints since it does not consider the relative value of each product. On the other hand, the proposed model, considers priority of each product. Therefore, priority of product is important as it justifies the product throughput of the organization and measures the performance of the system by optimizing in house and outsource quantity with maximum throughput. The proposed methodology described here has been analyzed and indicates several advantages:

- Laplace criterion decides the weightage of each product from the product contribution margin.
- The degree of importance for each of the products determines the profit obtained from the product-mix.

Some weakness of our proposed model might be:

- It is difficult to change the traditional method.
- It would take time to implement the approach.
- People may be reluctant to use it because they have to justify their own preferences, rather than simply saying yes or no.

It is, therefore, that the model can be widely applied for the industries will be of immense practical value. The model is valuable to contractor competing on contracts for more business. Further research is required into the application of this method in the multiple constrained resources.

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